

## THE SOFT POMERON

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## ABSTRACT

The soft pomeron successfully correlates a wide variety of data. Its properties seem rather simple: it couples to single quarks and its coupling factorises.

**1 Introduction**

The history of the soft pomeron goes back more than 35 years. In the 1960's a well-defined mathematical theory was developed, based on the idea of making angular momentum a complex variable, and there was a great deal of successful but very dirty phenomenology, but there was little or no understanding of what pomeron exchange is in physical terms.

In the 1970's there was rather little work on the subject; attention turned instead to hard processes.

In the 1980's data from higher energies revealed that actually the phenomenology is surprisingly clean. There were the beginnings of a crude physical understanding, based on nonperturbative gluon exchange, and there were several successful predictions.

Now, in the 1990's, HERA is providing important new data and reviving the interest in the soft pomeron. The hope is that this will lead to a fuller understanding, but it will surely be the 2000's before we have a good physical and theoretical understanding of what pomeron exchange actually *is*.

In studying the pomeron, it is particularly important to remember that high energy physics is *one* subject: we are much more likely to get an understanding if we correlate information from many reactions –  $ep$ ,  $\bar{p}p$ , ... Indeed, we cannot claim any success until we have done so. Mere parametrisation of data is of little use: we want a dynamical understanding. A superb fit with 20 parameters is much less use than a reasonable one with only 3, if we want to extract the physics message from the data.

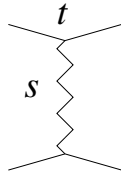


Figure 1: exchange of a family of particles

For these reasons, my philosophy is to explore how well one can do with the simplest assumptions. It is important, though, that they should be assumptions that do not conflict with known basic principles.

## 2. Complex angular momentum

A well-defined mathematical formalism, called Regge theory<sup>1</sup>, was developed more than 35 years ago to describe the exchanges of families of particles, for example the spin-one  $\rho$  together with its spin-3, spin-5,  $\dots$  excitations. Suppose that these exchanges are in the  $t$  channel: see figure 1. Consider the crossed channel, in which  $\sqrt{t}$  is the centre-of-mass energy, and  $\ell$  is the orbital angular momentum. The partial-wave amplitudes  $A(\ell, t)$  are then defined for  $\ell = 0, 1, 2, \dots$ . Continue them to complex values of  $\ell$  and introduce the “ $\rho$  trajectory”  $\alpha(t)$  defined such that

$$\alpha(m_\rho^2) = 1, \quad \alpha(m_{\rho_3}^2) = 3, \quad \alpha(m_{\rho_5}^2) = 5, \quad \dots \quad (1)$$

Experiment finds that  $\alpha(t)$  is linear in  $t$  and, within the errors, there are three other families whose trajectories all coincide with that of the  $\rho$ . These are the families  $\omega$ ,  $f$  and  $a$ : see figure 2. The significance of a trajectory  $\alpha(t)$  for a family of particles is that  $A(\ell, t)$  has a pole in the complex  $\ell$ -plane:

$$A(\ell, t) \sim \frac{1}{\ell - \alpha(t)} \quad (2a)$$

and this gives the amplitude of figure 1 a very simple high-energy behaviour in the channel where now  $\sqrt{s}$  is the centre-of-mass energy:

$$T(s, t) \sim \beta(t) s^{\alpha(t)} \xi_{\alpha(t)} \quad (2b)$$

That is, the amplitude varies with  $s$  as a simple power, and has a well-defined phase  $\xi_{\alpha(t)}$  that varies with the power. The function  $\beta(t)$  is not determined (it comes from whatever multiplies the pole (2b) in  $A(\ell, t)$ , but it is known to be real.

Unfortunately, it is known that  $A(\ell, t)$  does not only have poles in the complex  $\ell$ -plane: there are also branch points. A branch point at  $\ell = \alpha_C(t)$  contributes to the high-energy behaviour of  $T(s, t)$  the power  $s^{\alpha_C(t)}$ , divided by some function of  $\log s$  that depends on just what is the nature of the branch point.

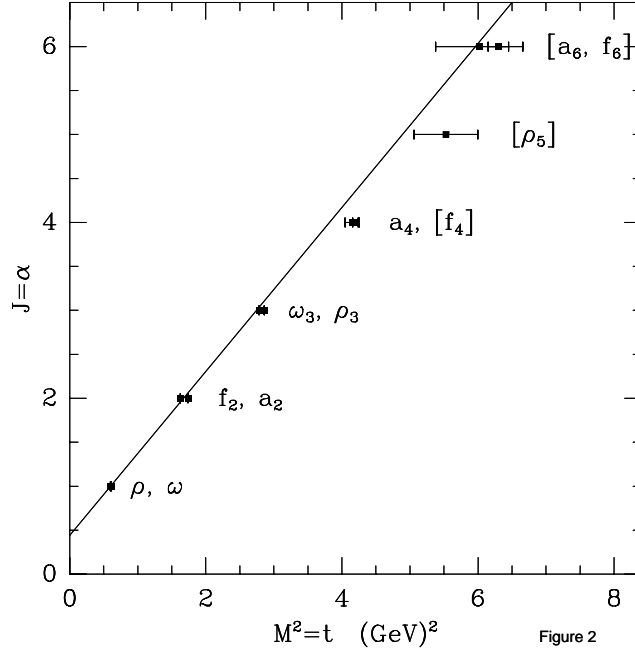


Figure 2: the  $\rho, \omega, f$  and  $a$  trajectories

So, to make a high-energy expansion of  $T(s, t)$ , look for the singularity in the complex  $\ell$ -plane with the largest  $\text{Re } \ell$ . This gives the leading power, together possibly with some log factor. The singularity with the next largest  $\text{Re } \ell$  gives the first nonleading power, and so on. For practical purposes, that is all: any other “background” should be negligible.

### 3. Total cross-sections and elastic scattering

From the optical theorem, the total cross-section is given by

$$\begin{aligned} \sigma^{\text{TOT}} &= \frac{1}{s} \text{Im } T(s, t=0) \\ &\sim s^{\alpha(0)-1} \end{aligned} \quad (3)$$

According to figure 2, for  $\rho, \omega, f, a$  exchange  $\alpha(0) \approx \frac{1}{2}$ , so these exchange contribute approximately the power  $1/\sqrt{s}$ . In order to describe data, we need also a term that rises slowly with  $s$ : see figure 3. The *simplest assumption* is that this corresponds also to a pole in the complex  $\ell$ -plane, and so also gives a simple power of  $s$ . In order to give a slowly-rising contribution to  $\sigma^{\text{TOT}}$ , it should be such that  $\alpha(0) = 1 + \epsilon_0$  with  $\epsilon_0$  a small positive number. We call this exchange pomeron exchange.

A complication is that, if we can have the exchange associated with a trajectory  $\alpha(t)$ , we can also have two or more such exchanges. For example, figure 4 shows

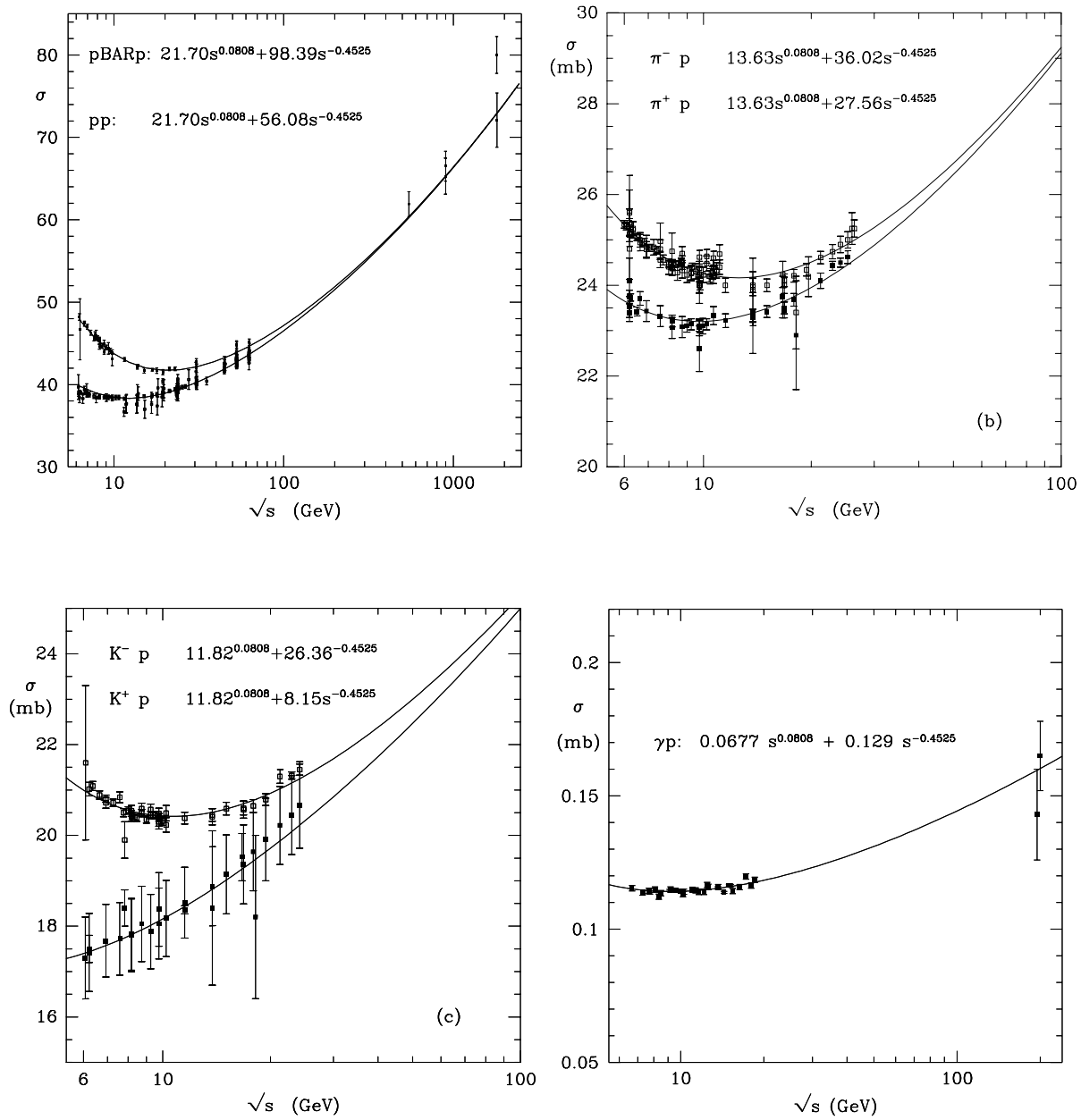


Figure 3: total cross-sections, with simple-power fits from reference<sup>22</sup>

double exchange, associated with the trajectories  $\alpha_1(t)$  and  $\alpha_2(t)$  (which may be the same). This is known<sup>1</sup> to give a branch point in the complex  $\ell$ -plane, whose position is

$$\ell = \alpha_C(t)$$

$$\alpha_C(0) = \alpha_1(0) + \alpha_2(0) \quad (4)$$

So the exchange of two pomerons contributes to  $\sigma^{\text{TOT}}$  a term  $s^{2\epsilon_0}$ , divided by some function of  $\log s$  and multiplied by some constant which we cannot calculate, though

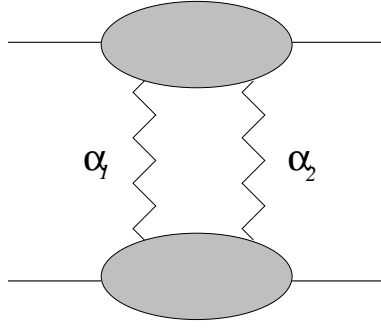


Figure 4: double exchange

we know that it is negative. The *simplest assumption* is that this constant is small. Then the sum of the exchanges  $P + PP$  will behave as an effective power  $s^\epsilon$ , with  $\epsilon$  just a little less than  $\epsilon_0$  and decreasing slowly with  $s$  as  $s$  increases. According to the fits in figure 3, experiment finds  $\epsilon \approx 0.08$ .

In the fits of figure 3, the ratio of the strengths of pomeron exchange in  $\pi p$  and  $pp$  or  $\bar{p}p$  scattering is  $13.6/21.7 \approx 2/3$ . This is an indication that the pomeron couples to single valence quarks in a hadron, and is called the “additive-quark rule”. The *simplest assumption*<sup>3</sup> is that its coupling to a quark is like that of a photon, with a Dirac  $\gamma$  matrix times a constant  $\beta_0$ . Then the contribution from pomeron exchange to the quark-quark elastic scattering amplitude is

$$\gamma \cdot \gamma \beta_0^2 s^{\alpha(t)-1} \left( -e^{-\frac{1}{2}i\pi\alpha(t)} \right) \quad (5)$$

The last factor is the phase factor  $\xi_{\alpha(t)}$  of (2b) for the case of charge parity  $C = +1$  exchange; the inclusion of this phase is what makes pomeron exchange different from photon exchange. For  $pp$  or  $\bar{p}p$  scattering, we need to take account of the wave function of the quarks in the nucleon. Just as for photon exchange, we do this by introducing two Dirac elastic form factors,  $F_1(t)$  and  $F_2(t)$ . These have been measured in  $ep$  scattering, but there it is the photon that is exchanged, and it has  $C = -1$ . The *simplest assumption*, which works better than can really be understood, is that the  $C = +1$  and  $C = -1$  form factors are equal. Since pomeron exchange is isospin 0, this means that we use the sum of the proton and neutron form factors measured in elastic electron scattering. For the case of  $F_2$ , this sum is small — at  $t = 0$  it is equal to the sum of the anomalous magnetic moments of  $p$  and  $n$ , which is small. The presence of an  $F_2$  term would correspond to nucleon helicity flip, which has long been known to be small for pomeron exchange; it is interesting that this can be linked to the anomalous moments<sup>3</sup>. For the neutron, the form factor  $F_1$  is by definition 0 at  $t = 0$ , and it is known to remain small away from  $t = 0$ , so the form factor  $F_1(t)$  that we need is just the proton form factor  $F_1(t)$  measured in elastic  $ep$  scattering. The related Sachs form factors  $G_E(t)$  and  $G_M(t)$  are found to be proportional to each other and of dipole form; the data for

these correspond to

$$F_1(t) = \frac{4m^2 - 2.8t}{4M^2 - t} \left( \frac{1}{1 - t/0.7} \right)^2 \quad (6)$$

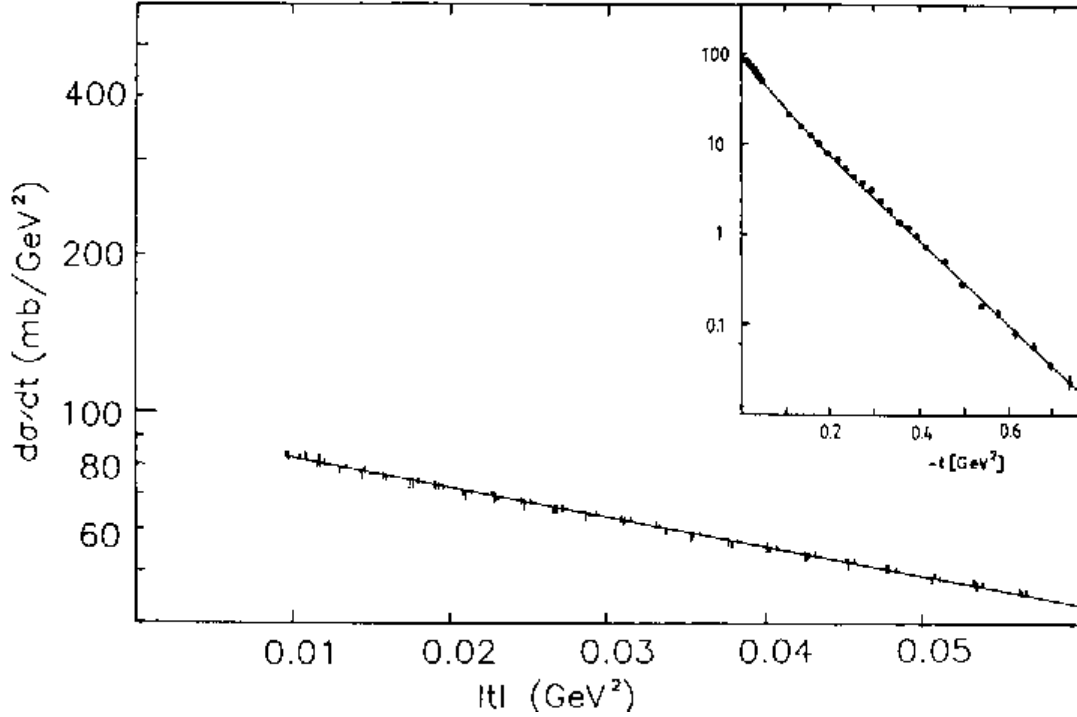


Figure 5:  $pp$  elastic scattering at  $\sqrt{s}=53$  GeV

Introducing the *simplest assumption* that the pomeron trajectory  $\alpha(t)$  is linear in  $t$ , though allowing for the possibility that it has a different slope  $\alpha'$  from the trajectories shown in figure 2, we find that single pomeron exchange contributes to elastic  $pp$  or  $\bar{p}p$  scattering

$$\frac{d\sigma}{dt} = \frac{[3\beta_0 F_1(t)]^4}{4\pi} (\alpha' s)^{2\epsilon_0 - 2\alpha'|t|} \quad (7)$$

The value of  $\alpha'$  may be determined by fitting this to the highly accurate CERN ISR small- $t$  data at  $\sqrt{s}=53$  GeV, shown in figure 5. The inset in this figure shows that then the form (7) fits extremely well to the data at larger  $t$ . This is a nontrivial check that the assumption about  $F_1(t)$  is surprisingly correct; as it comes into (7) raised to the fourth power the fit is rather sensitive to it. The form (7) is found to agree well with data at all energies<sup>4</sup>, including the Tevatron data at  $\sqrt{s} = 1800$  GeV. It correctly predicted that the forward peak at this energy would be rather steeper. According to (7), if one fits to  $e^{-b|t|}$  then when the energy is increased by

a factor  $R$  the slope  $b$  decreases by an amount  $\alpha' \log R$ , which is about 3.5 when the energy increases from ISR to Tevatron values. Notice, though, that a fit with  $e^{-b|t|}$  can only be local, unless one allows  $b$  to vary with  $t$ .

Single-pomeron exchange is not the whole story. There are also nonleading exchanges, in particular  $\rho, \omega, f, a$ , though these have become unimportant when the energy is as high as 53 GeV. What cannot be ignored is the exchange of two pomerons. While we do not know how large is the contribution from this, we do know about its general features: see figure 6. It is flatter than single-pomeron exchange, and as  $s$  increases it steepens half as quickly. But at  $t = 0$  it rises twice as fast as single-pomeron exchange. So, as  $s$  increases the point where the two are equal moves to lower and lower  $t$ . One consequence of this is that the shape of the differential cross-section, as a function of  $t$ , changes with increasing energy. It happens that, at Tevatron energy, the two contributions combine in such a way that a fit  $e^{-b|t|}$  with  $b$  independent of  $t$  is quite good<sup>5</sup>, though this is not true at either lower or higher energies.

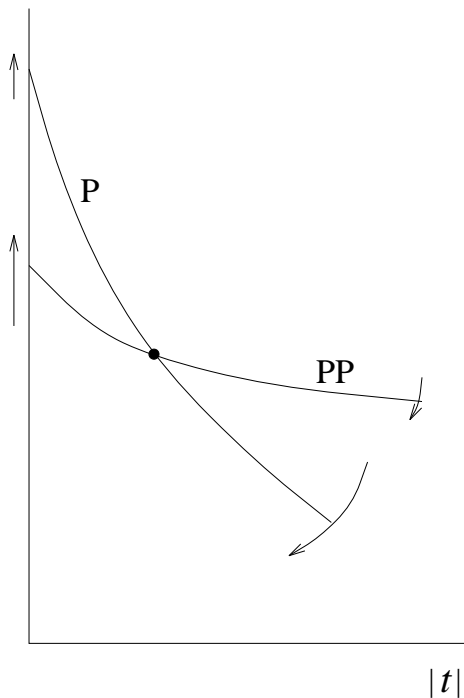


Figure 6: contributions to  $\frac{d\sigma}{dt}$  from single and double pomeron exchange. The arrows indicate how they change as the energy increases.

Having established that, for  $t < 0$ , the pomeron trajectory is

$$\alpha(t) = \epsilon_0 + 0.25t \quad (8)$$

with  $\epsilon_0$  between 0.8 and 0.9, we may extrapolate it to positive  $t$ . The *simplest*

*assumption* is that it remains straight, and then  $\alpha(t) = 2$  at a value of  $t$  just less than 4 GeV<sup>2</sup>. This leads us to predict that there should be a  $2^{++}$  particle with a mass just less than 2 GeV. Since theoretical prejudice leads to the belief that pomeron exchange is gluon exchange, this particle would be a glueball. It is interesting that the WA91 experiment<sup>6</sup> has reported a “ $2^{++}$  glueball candidate” at just the right mass.

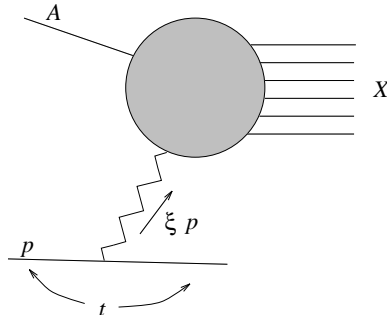


Figure 7: diffraction dissociation

## 4. Diffraction dissociation

Figure 7 shows diffraction dissociation: some projectile  $A$  hits a proton and breaks up into a system  $X$  of hadrons, while the proton survives and retains almost all its momentum. The projectile  $A$  can be any particle, for example another proton, or a  $\gamma$  or  $\gamma^*$ . The fractional momentum loss  $\xi$  of the target proton should be less than a few percent. In this case it is a matter of simple kinematics to understand that the final state can have no other particle close in rapidity to the target proton, so these events are “large-rapidity-gap” events. The magnitude of  $\xi$  may be calculated from the invariant mass of the system  $X$  of fragments of the projectile particle:  $\xi = M_X^2/s$ . Instead of  $\xi$ , the notation  $x_P$  is often used.

If  $\xi$  is small enough, the exchanged object in figure 7 should be the pomeron. If pomeron exchange is described by a simple pole in the complex  $\ell$ -plane, it should factorise:

$$\begin{aligned} \frac{d^2 \sigma^{Ap}}{dt d\xi} &= F_{P/p}(\xi, t) \sigma^{PA}(M_X^2, t) \\ F_{P/p} &= \frac{9\beta_0^2 [F_1(t)]^2}{4\pi} \xi^{1-2\alpha(t)} \end{aligned} \quad (9)$$

Even if there is a glueball associated with the pomeron trajectory near  $t = 4 \text{ GeV}^2$ , when it is exchanged near  $t = 0$  the pomeron cannot be said to be a particle. Nevertheless, the factorisation (9) makes pomeron exchange very similar to particle exchange: the factor  $\sigma^{PA}(M_X^2, t)$  may be thought of as the cross-section for



pomeron- $A$  scattering. When its subenergy  $M_X$  is large, it should have much the same power behaviour as the hadron-hadron total cross-sections shown in figure 3:

$$\sigma^{PA}(M_X^2, t) \sim u(t)(M_X^2)^{0.08} + v(t)(M_X^2)^{-0.45} \quad (10)$$

But there are complications: the zigzag line in figure 7 may not be the pomeron. Simple pomeron exchange may be contaminated in two ways. If  $\xi$  is not small enough, one must add in a contribution from  $\rho, \omega, f, a$  exchange, or even  $\pi$  exchange when  $t$  is close to 0. That is, these exchanges can also result in large rapidity gaps, though as they correspond to smaller powers of  $1/\xi$  than pomeron exchange, they become relatively less important as  $\xi$  decreases. If one integrates (9) down to some fixed  $M_X^2$ , the resulting cross-section for diffraction dissociation behaves as  $s^{2\epsilon_0}$ , and so unless something else intervenes it would become larger than the total cross-section<sup>7</sup>. As  $s$  increases at fixed  $M_X^2$ , one is probing larger and larger values of  $1/\xi$ , so one expects that the same happens as in the total cross-section: the exchange of two pomerons becomes important and moderates the rising contribution from single exchange. But the *simplest assumption* is that this matters only at very small  $\xi$ .

Notice that the theory leads us to expect that adding these other exchanges should give us all the nonleading powers of  $1/\xi$ : there should be no other appreciable “background”. Note also that adding in the other exchanges will surely break factorisation. Further, it is likely that, even though  $f$  exchange, in particular, gives a nonleading power of  $1/\xi$ , it may be numerically important down to quite small  $\xi$ . This certainly seems to be true for diffraction dissociation in  $pp$  or  $\bar{p}p$  collisions<sup>8</sup>. Donnachie and I parametrised<sup>9</sup> the ISR data in the simplest manner: we included  $f$  exchange simply by multiplying the pomeron-exchange contribution (9) by the factor

$$1 + 2C\xi^a(t) \cos \tfrac{1}{2}\pi a(t) + C^2\xi^{2a(t)} \\ a(t) = \alpha_P(t) - \alpha_f(t) = 0.64 - 0.68t \quad (11)$$

The  $\xi^{2a(t)}$  term corresponds to the pomeron in figure 7 being replaced with an  $f$ , while the  $\xi^{a(t)}$  term is interference between pomeron and  $f$  exchange. We found that  $C$  is large, about 8, which means that at  $t = 0$  the factor is greater than 2 even when  $\xi$  is as small as 0.02. There is no reason to suppose that it is actually correct to use a simple factor such as (11), and the magnitude of the effect could be substantially different for different projectiles, such as  $\gamma^*$ .

The case where the projectile  $A$  in figure 7 is a  $\gamma^*$  is what is studied in the “diffractive events” at HERA. In this case, a factorising single-pomeron exchange would give a factorising contribution to the proton structure function from very-fast-proton events:

$$\frac{d^2 F_2^{\text{DIFFRACTIVE}}}{dt d\xi} = F_{P/p}(\xi, t) F_2^{\text{POM}}(\beta, Q^2, t) \quad (12)$$

where  $\beta = x/\xi$ . Here,  $F_2^{\text{POM}}$  may be thought of as the “structure function of the pomeron”: it is defined if the pomeron is a simple pole in the complex  $\ell$ -plane and so gives a factorising contribution, even though it is not a particle.

According to what I have said, one has to worry about possible contamination, particularly from  $f$  exchange. This is likely to be important if  $\xi$  is not small enough. But the value of  $\xi$  below which one can forget it may well be  $\beta$ -dependent. If the structure function of the  $f$  is much larger at small  $\beta$  than that of the pomeron, then it might give appreciable contamination at small  $\beta$  even when  $\xi$  is rather small.

Our theoretical understanding of the pomeron structure function is so far very rudimentary, though it did allow the prediction<sup>10</sup> that surprisingly-large fraction of small- $x$  events at HERA would have a very fast proton in the final state. This prediction used the simplest model, which exploits the similarity between the pomeron and a photon, though with the important difference that the pomeron does not couple to a conserved current. This leads to a quark structure functions

$$\beta q^{\text{POM}}(\beta) = C\beta(1 - \beta) \quad (13)$$

with  $C \approx 0.25$  for each light quark and antiquark. A similar form results<sup>11</sup> from modelling pomeron exchange as two-gluon exchange. Just as for the case of the photon structure function, one has to add in a term that is important at small  $\beta$  and behaves like  $\beta^{-\epsilon_0}$ , with  $\epsilon_0 = 0.08$ , or maybe larger. This is certainly only the crudest model, and it leaves many obvious questions. How does  $q^{\text{POM}}(\beta)$  evolve<sup>12</sup> with  $Q^2$ ? How large is the charm structure function? And what is the gluon structure function? We have no model for the pomeron’s gluon structure function, and cannot even tell how large it should be — as the pomeron is not a particle, there is no momentum sum rule.

## 5. Electroproduction of vector mesons

So far, our theoretical understanding of the soft pomeron in terms of QCD is only very crude. There is a consensus that pomeron exchange is gluon exchange and that the soft pomeron is nonperturbative and so the gluons are not perturbative. Of course, it is this which hinders any clean calculation.

The gluon is confined, which means that its propagator  $D(k^2)$  should have the perturbative  $k^2 = 0$  pole removed by nonperturbative effects. This means that in the ratio

$$\mu^2 = \frac{(\int_{-\infty}^0 dk^2 \, 2k^2 D^2(k^2))}{(\int_{-\infty}^0 dk^2 \, D^2(k^2))} \quad (12)$$

the integrals in both the numerator and the denominator should converge at  $k^2 = 0$ . Because confinement is a nonperturbative effect, and because the typical nonperturbative scale is 1 GeV, we expect the mass  $\mu$  defined by (12) to be about 1 GeV.

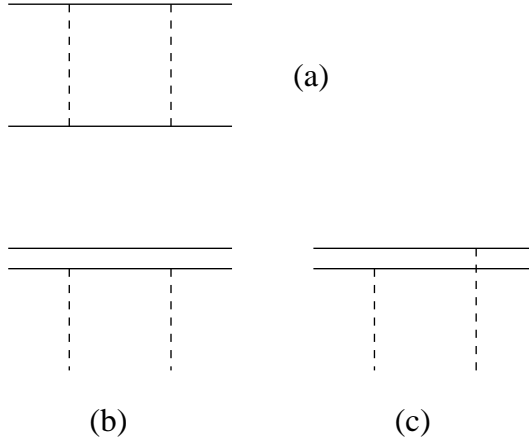


Figure 8: (a) two-gluon between quarks; (b) and (c) couplings of two gluons to the quarks in a pion

In order to model pomeron exchange by gluon exchange, we need at least two gluons to reproduce the colour-singlet isoscalar nature of the pomeron. the simplest model for pomeron exchange between quarks is thus figure 8a. At  $t = 0$  this diagram is just a constant times the denominator of (12), so the model makes sense only because of confinement. Crude as it is, the model already has some success<sup>13</sup> in explaining observed properties of the soft pomeron: one finds from it that the two gluons couple to each quark like a single photon-like object, with Dirac matrix  $\gamma$ . Further, when the two gluons couple to the quarks in a hadron, one can understand why they prefer to couple to the same quark: in the case of a pion the coupling of figure 8b is much larger than that of figure 8c because the pion radius  $R$  satisfies  $\mu^2 R^2 \ll 1$ . Thus the additive-quark rule may be understood.

One may refine this simple model<sup>14</sup> by allowing the exchange of more than two gluons, but the problem of calculating the energy dependence  $s^{\epsilon_0}$  of pomeron exchange is still too difficult; this factor has to be put in by hand.

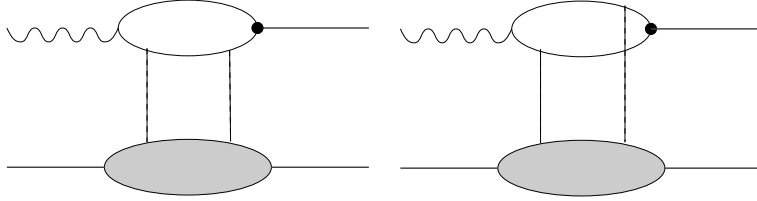


Figure 9: simple model for  $\gamma^* p \rightarrow \rho p$

A good test of the simple model is exclusive  $\rho$  electroproduction,  $\gamma^* p \rightarrow \rho p$ . Apparently different approaches to this process actually share common key features: see figure 9. At the top of each graph is a quark loop that couples the  $\gamma^*$  to the  $\rho$ . There

are two different ways in which the gluons couple; the additive-quark rule would make the first graph dominate for a real photon, but as  $Q^2$  increases the second graph becomes more and more important. It tends to cancel the first graph: this is called ‘colour transparency’. As for the bottom bubble in the graphs, one approach is to pretend<sup>15</sup> that it is the gluon structure function of the proton, though in fact it cannot exactly be that and indeed the assumption could be altogether wrong<sup>16</sup>. Using the gluon structure function leads to a rapid rise with increasing energy  $W$ .

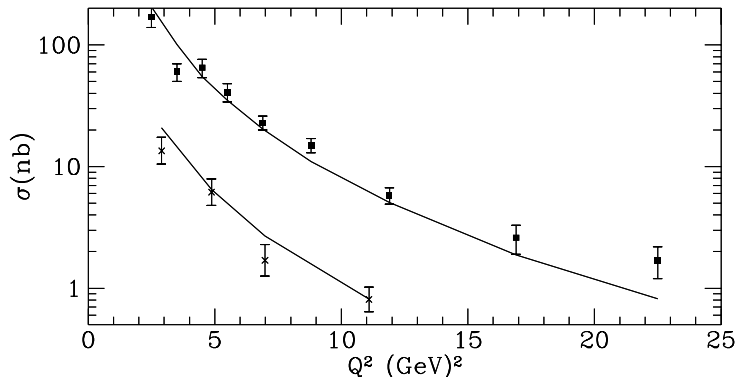


Figure 10: NMC data<sup>17</sup> for  $\gamma^*p \rightarrow \rho p$  and  $\gamma^*p \rightarrow \phi p$ , with calculated curves from reference 16

A simpler model<sup>18</sup> is to replace the bottom bubble with the same simple coupling to a quark as in figure 8. Then the  $W$  dependence has to be put in by hand. This model is surprisingly successful in its agreement with low-energy data: see figure 10, which includes also  $\gamma^*p \rightarrow \phi p$ . The calculated curves shown in figure 10 also make a simple assumption about the  $\rho$  vertex: that it is strongly peaked such that the two quarks at the vertex prefer to share equally the momentum of the  $\rho$ . A test of this is the consequence for the  $\rho$  polarisation (which should be equal to that of the  $\gamma^*$ ). For  $Q^2 \gg m_\rho^2$  the longitudinal amplitude is proportional to  $f_\rho/Q^3$  and the transverse amplitude to  $f_\rho m_\rho/Q^4$ . The ratio of the amplitudes is predicted to be about 2 for  $Q^2 = 5 \text{ GeV}^2$ , rising to 8 at  $Q^2 = 20$ , which seems to be in agreement with the low-energy data. Because of the extra factor  $m/Q$  in the transverse amplitude, for heavier vector mesons we expect to need rather larger  $Q^2$  before the longitudinal production dominates. Thus for  $\gamma^*p \rightarrow J/\psi p$ , the simple model predicts that we have to go to  $Q^2=100$  before the longitudinal amplitude is twice as large as the transverse. Nevertheless the transverse amplitude is big if the coupling of the (nonperturbative) gluons to the quarks is flavour-blind:  $J/\psi$  production overtakes  $\rho$  production at around  $Q^2=10$ . How much these predictions depend on the explicit assumptions about the vertex is not understood.

## 6. Conclusions

The soft pomeron successfully correlates a variety of  $Q^2 = 0$  data. Its properties are probably simple – it seems to couple to single quarks in a factorising manner, indicating that it is associated with a simple pole in the complex  $\ell$ -plane.

Nevertheless, there are some big surprises in the HERA data. The cross-section for quasi-elastic  $J/\psi$  photoproduction,  $\gamma p \rightarrow J/\psi p$ , rises more rapidly with energy than soft-pomeron exchange would have predicted, and the proton structure function  $F_2$  rises spectacularly rapidly as  $x$  becomes very small. An immediate explanation that comes to mind is that one is seeing the effects of the perturbative BFKL pomeron<sup>19</sup>, but this is unlikely to be correct<sup>20</sup>. There are several other candidate explanations<sup>21</sup>, but no general agreement about what is the right one. As was said by Uri Maor at the recent meeting in Eilat:

“One of the reasons it is a beautiful subject is that there are lots of things we don’t understand”.

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